

# Negative refraction in a prism made of stacked subwavelength hole arrays

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**Abstract:** Metamaterial structures are artificial materials that show unconventional electromagnetic properties such as negative refraction index, perfect lenses, and invisibility. However, losses are one of the big challenges to be surpassed in order to design practical devices at optical wavelengths. Here we report negative refraction in a prism engineered by stacked sub-wavelength hole arrays. These structures exhibit inherently an extraordinary optical transmission which could offer a solution to the problem of losses at optical wavelengths. It is shown the possibility to obtain negative indices of refraction starting from near to zero values. Our work demonstrates by a direct experiment the feasibility of engineering negative refraction by just drilling sub-wavelength holes in metallic plates and stacking them.

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## 1. Introduction

The pursuit of low-loss metamaterials exhibiting Negative Refractive Index (NRI) at optical frequencies is today one of the most important challenges [1] since the publication of Pendry's perfect lens paper [2] and the first experimental confirmation of NRI for microwaves by Smith's group [3]. In Veselago's seminal paper [4] it was shown how a medium having simultaneously negative permittivity and permeability exhibits unconventional properties such as a NRI leading to the opposite Snell's refraction law at the interface between a standard and a NRI medium. However, all of these media are artificial materials with a very recent exception, see [5]. The promise of the availability of these metamaterials at optical frequencies have refreshed the research in classical electrodynamics by introducing highly unconventional properties that can lead to a new class of devices like perfect lenses<sup>2</sup> and invisibility cloaks [6]. The phenomenon of negative refraction was demonstrated initially in dielectric photonic crystals, i.e., in inhomogeneous periodic media with a lattice constant comparable to the wavelength [7].

For NRI to happen, both the permittivity and permeability must be simultaneously negative [2]. Pendry's group had previously shown the feasibility to fabricate an artificial material with negative permittivity using a lattice of thin metal wires [8], and they had also reported how to obtain a negative magnetic medium from a lattice of "split rings resonators" [9]. Smith et al. combined both ideas and built a structure with simultaneous negative  $\epsilon$  and  $\mu$  [10], showing negative refraction for the first time by using a prism [3]. Since then, left-handed metamaterials have been achieved for frequencies in the microwave range from splitting resonators components and the complementary particle [11]. Very recently, the possibility of NRI materials at optical frequencies has been opened by the fabrication of metamaterials using localized plasmonic resonances such as those happening in metal nanorod doublet arrays [12] and in a pair of subwavelength hole arrays drilled in very thin metallic plates [13]. However, losses are still an important challenge in these structures. Also, the possibility of having NRI in subwavelength hole array structures has been demonstrated in the microwave range [14] where the losses are inherently much lower since metals behave nearly as Perfect Conductors in this range. Losses could be further reduced thanks to an innovative approach based upon a double periodicity in the subwavelength hole array allowing to increase the hole density and to operate in the Fresnel zone of the structure [15].

In this work, we report a straightforward pure geometrical and experimental demonstration of a low-loss negative refraction structure in the low terahertz band by using a metamaterial prism made by just stacking sub-wavelength metallic hole arrays sandwiched in air, see Fig. 1. It should be noted that indirect phase and interferometric techniques have been previously used to claim left-handed propagation in these stacked hole arrays structures [14-16]. In properly engineered subwavelength hole arrays [14,15], low losses come up as a natural consequence of Extraordinary Optical Transmission (EOT) and can open the way to a novel design procedure of low-loss negative refraction media at microwave, terahertz, and even optical wavelengths. EOT consists in high transmittance peaks in the hole cut-off region and is caused by surface plasmons in optics [17], but it also appears at microwaves [18]

although metals here are nearly perfect conductors and therefore do not support plasmons. The explanation is that perforated metallic films support surface waves which somehow mimic surface plasmons [19].

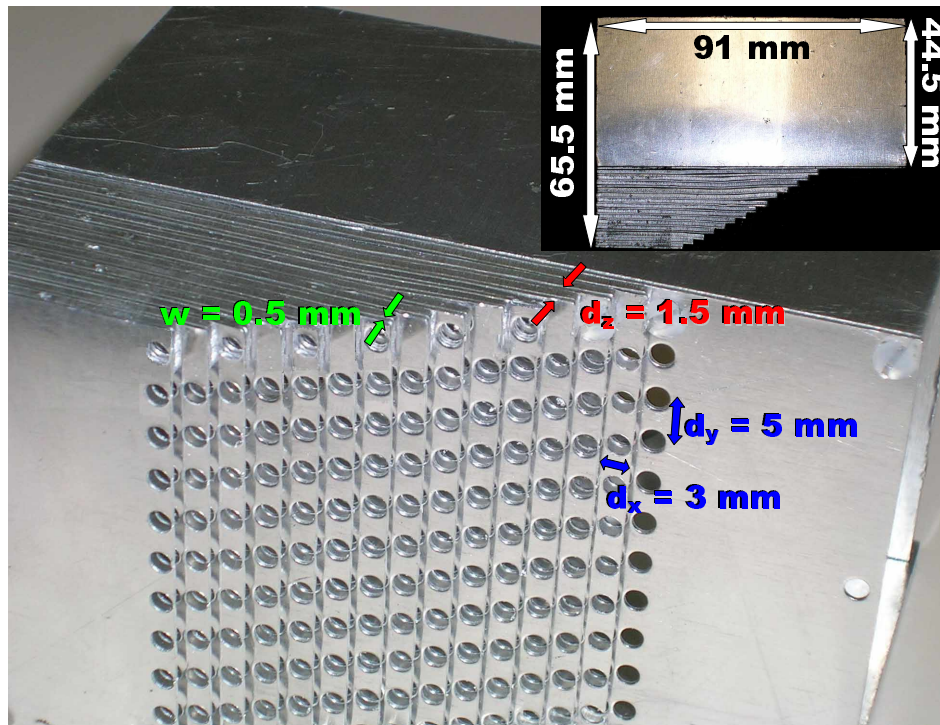


Fig. 1. Details of the prism structure with parameters,  $a = 2.5$  mm,  $d_x = 3$  mm,  $d_y = 5$  mm, and  $d_z = 1.5$  mm and experimental set-up as an inset. The angle of the prism is 26.6 deg.

## 2. Design of the Prism made of Stacked Subwavelength Hole Arrays.

Therefore, by drilling holes in a perfectly conducting material surface waves can be engineered and EOT achieved. Furthermore, as we have shown in a previous work, by periodically stacking subwavelength hole arrays having EOT, a left-handed inner propagation can be created as well [14].

Only at small separations between the plates (much smaller than the resonant wavelength) the coupling leading to left-handed propagation happens so that this kind of structure can be considered as a metamaterial.

With the double period, see Fig. 1, two interesting features are obtained: first, grating lobes appear far from the EOT band thanks to the fact that for an electromagnetic wave impinging normally on a periodic structure, in the far field, the scattered waves have to obey the following geometrical identity to interfere constructively,

$$d_x \cdot \sin\theta = m \cdot \lambda$$

being  $d_x$  the horizontal structure lattice,  $\theta$  is the angle of emergence of the scattered wave with respect to the incident one,  $\lambda$  is the wavelength of the interacting electromagnetic wave, and  $m$  is an integer. Therefore, due to the fact that the lattice along x-axis (horizontal) is small, the previous identity is only fulfilled for the case  $m = 0$ , i.e., no grating lobes appear near the EOT band.

And second, the number of illuminated holes increases [15]. In the experiment two metallic plates are introduced above and below, which, due to mirror effects increase the effective number of illuminated holes. The longitudinal lattice,  $d_z$ , between the hole array plates is  $d_z = 1.5$  mm. Finally, to achieve the desired prism structure, the number of periods along  $x$  is gradually removed, see Fig. 1.

As mentioned, the most simple and direct experimental way to demonstrate negative refraction in a given metamaterial is by using a prism which is governed by Snell's law:  $n_1 \cdot \sin(\theta_1) = n_2 \cdot \sin(\theta_2)$ , being  $n_1$  and  $n_2$  the refractive indices of the metamaterial prism and of air. The involved angles are measured from the interface normal. Contrary to ordinary materials, refracted rays emerge on the same side of the normal in NRI metamaterials which constitutes an unambiguous proof of the refractive index sign.

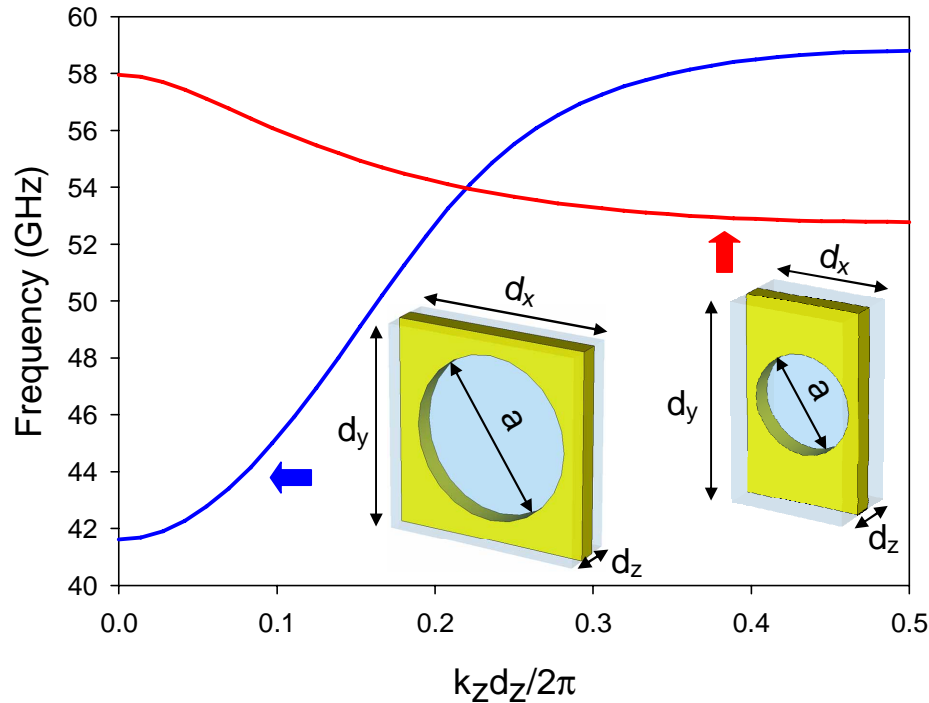


Fig. 2. Simulated dispersion diagrams. Stacked sub-wavelength hole arrays with  $a = 2.5$  mm,  $d_x = 3$  mm,  $d_y = 5$  mm, and  $d_z = 1.5$  mm (solid red trace), and propagating hole arrays with  $a = 4$  mm,  $d_x = 5$  mm,  $d_y = 5$  mm, and  $d_z = 1.5$  mm (solid blue trace).

In order to gain insight, numerical simulations of the dispersion diagram and the electric field evolution inside the metamaterial were carried out based on a finite-difference time domain approach. The dispersion diagrams for several structures of stacked hole arrays are displayed in Fig. 2. There it is shown the dispersion diagram corresponding to the structure with the aforementioned experimental parameters, and the dispersion diagram for a structure with propagating holes. Note that in order to hold the hole diameter ( $a = 4$  mm) the small period in the double periodic structure has been increased from 3 mm up to 5 mm leading to a square structure. The first band of the sub-wavelength hole array structure (solid red trace) emerges around the EOT frequency and it clearly shows a negative slope, i.e. phase velocity opposite to the group velocity. For the holes in propagation (solid blue trace), the first band shows a positive slope, i.e. it becomes right-handed, and the left-handed effect disappears.

Subsequently, we have considered the evolution of the vertical electric field in a prism made of stacked subwavelength hole arrays, with the aforementioned parameters, along the  $x$ - $z$  cutting plane (where the radiation pattern is recorded) at the frequency of 53.5 GHz where

simulations give an index of refraction  $n = -1$ , see Fig. 3(a). In Fig. 3(b) the case of a prism made of stacked propagating hole arrays is shown. It is clear that the power flow for subwavelength holes emerges breaking the usual direction of Snell's law as it can be expected in a negative index of refraction metamaterial, see Fig. 3(a) and the supporting video files. This is also in agreement with the dispersion diagrams presented in Fig. 2. For the stacked propagating hole arrays, the behaviour is the usual one of right handed materials.

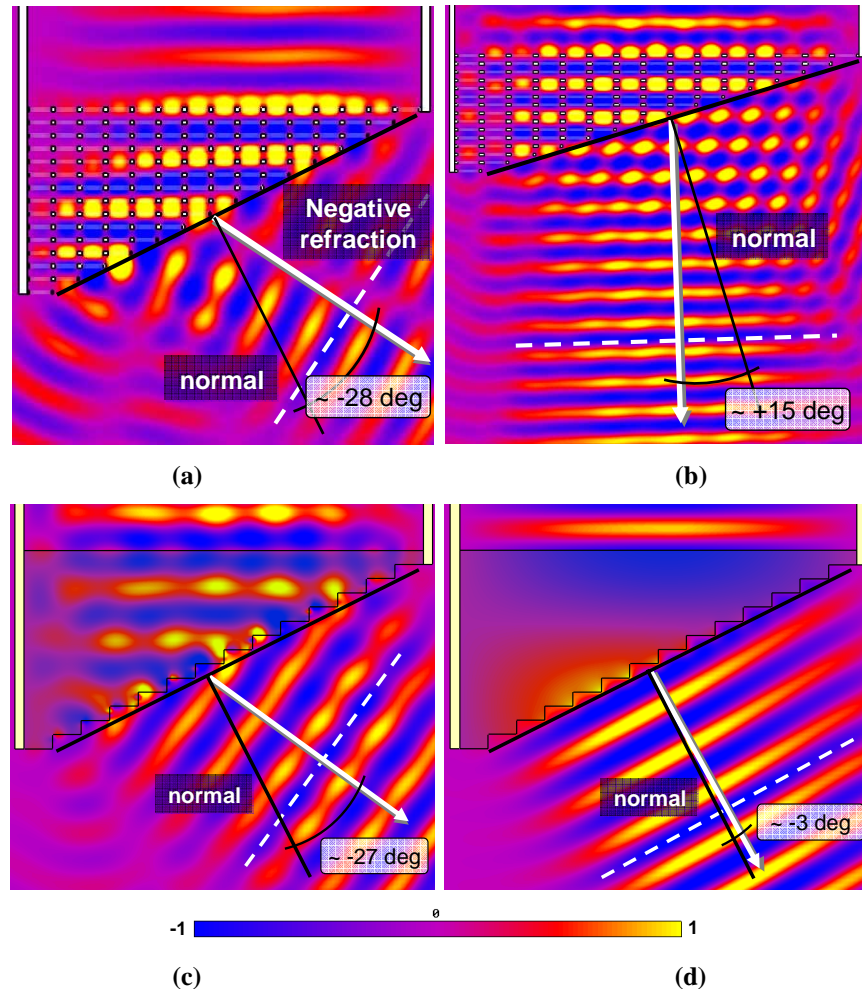


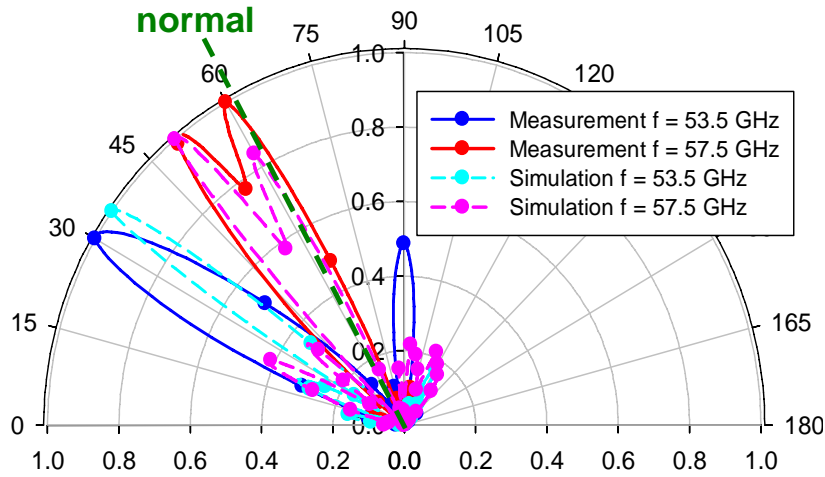
Fig. 3. Simulated vertical electric field evolution from the inner of the prism towards the surrounding air for stacked subwavelength hole arrays (a) and propagating stacked hole arrays (b). The same for homogenized structure at 53.5 GHz (c) and for 57.5 GHz (d).

Supporting video files are attached for a real-time visualisation of the electric field evolution along a prism made of stacked subwavelength hole arrays exhibiting a negative index of refraction.

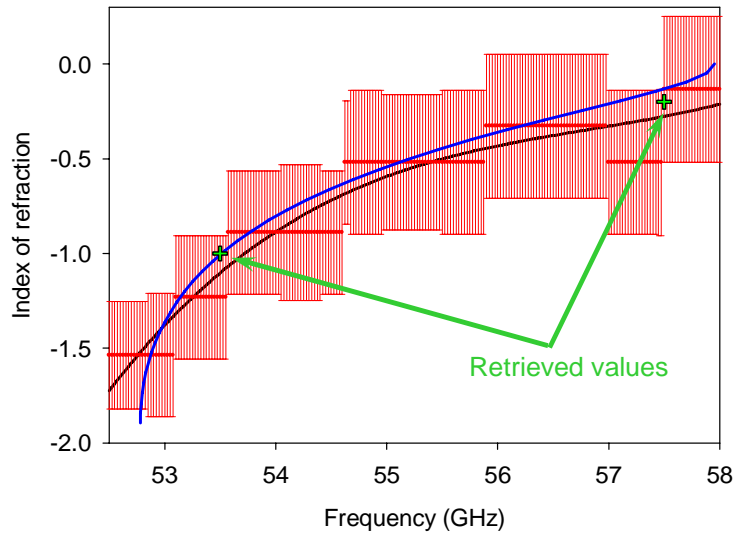
### 3. Experimental results.

Employing an AB Millimetre<sup>TM</sup> quasi-optical vector network analyzer the angular evolution of the received radiated power is recorded for the subwavelength holes prism. The distance between the prism and the receiver is 400 mm which is in the limit of the far field or

Fraunhofer zone. The measurement frequency range extends from 40 to 70 GHz (wavelength from 4.29 mm up to 7.50 mm).



(a)



(b)

Fig. 4. Measurements of the metamaterial prism constructed by stacked subwavelength hole arrays. (a) Angular power distribution for two different frequencies:  $f = 53.5$  GHz (solid blue) and  $f = 57.5$  GHz (solid red). The spots correspond to the measurement results whereas the curves are a smoothed spline interpolation. Also shown the simulation results for comparison purposes (dashed lines). (b) Evolution of refractive index with the frequency simulation and measurements. Red spots are the experimental points along with the corresponding error bars, the black curve is the polynomial interpolation (order 3) of the experimental data and the blue one is the simulation result.

The recorded normalized amplitude, Fig. 4(a), shows for two different frequencies (53.5 and 57.5 GHz) that the radiation pattern emerges on the same side of the normal, which proves unambiguously the fact that our proposed structure exhibits a negative index of refraction. It is noticeable that as the frequency increases the peak approaches the normal which indicates that the refractive index is dispersive, i.e., it depends on the frequency. In this

particular case, the index tends to zero but still remains negative as it can be easily derived from Snell's refraction law. This phenomenon is presented in Fig. 4(b) where the experimentally obtained values show an excellent agreement with the simulations results. Furthermore, using a retrieval method [20], particularized in the EOT band, we have obtained a relative dielectric permittivity of  $\epsilon_r = -1.2$  at 53.5 GHz and  $\epsilon_r = -0.2$  at 57.5 GHz whereas the obtained relative magnetic permeability is  $\mu_r = -0.8$  and  $\mu_r = -0.1$  respectively. These values lead to refractive indices of  $n = -1.0$  at 53.5 GHz and  $n = -0.2$  at 57.5 GHz which are in agreement with the experimental results on the refraction index, see Fig. 4, and the homogenized simulations presented in Figs. 3(c)-3(d).

It is worth stressing that EOT through subwavelength hole arrays was originally achieved in the visible spectrum [17], and afterwards demonstrated in the microwave regime provided a minimal number of interacting holes is present [18]. Now, by just stacking subwavelength hole arrays, left-handed metamaterials have been already achieved for frequencies in the microwave range [14,15], being now the challenge to build left-handed metamaterials that work at visible wavelengths.

The scaling up of left-handed structures based on split ring resonators to the optical regime is far from trivial [1]. It is now reasonable to expect that left-handed metamaterials showing low losses due to EOT can be achieved in the optical regime by the proper stacking of perforated metallic plates. Here, the role of the EOT phenomenon in sub-wavelength hole arrays is essential in order to have an important useful power level at the output of the device and to provide the necessary shunt inductance to cause left-handed propagation when the plates are close enough.

## 5. Conclusion

The employed double periodic hole array structure exhibits anisotropy for the impinging electromagnetic wave polarization and angle of incidence. This fact may be regarded as a limitation but, simultaneously, opens the way to future applications and theoretical studies. The experimental results obtained in our novel prototype could lead to a new class of practical devices both in the microwave, the terahertz, and in the optical range. Further experiments and theoretical analysis are needed to grasp the full implications of these findings and to obtain practical devices.

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