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# The role of dipolar interactions for the determination of intrinsic switching field distributions

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The $\Delta H(M,\Delta M)$ method and its ability to determine intrinsic switching field distributions of perpendicular recording media are numerically studied. It is found that the presence of dipolar interactions in the range of typical recording media substantially enhances the reliability of the $\Delta H(M,\Delta M)$ method. In addition, a strong correlation is observed between the precision of this method and a self-consistency check of the data sets, which is based upon a simple redundancy measure. This suggests that the latter can be utilized as an efficient criterion to decide if a complete data analysis is warranted or not. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2938695]

For magnetic recording media used in state-of-the-art storage applications such as hard disks drives, the intrinsic switching field distribution $D(H_S)$ of the media grains is one of the most crucial properties defining the recording quality.[1] In general, each grain is characterized by an intrinsic switching field $H_S$, which is a local material property. Because grains interact with each other by means of exchange and dipolar interactions, $D(H_S)$ is not easily accessible in macroscopic measurements. This is especially true for perpendicular recording media (PRM) due to the strength of the interactions, which is much larger than in the previously used longitudinal recording media.[2] Therefore, though several methodologies have been developed to determine $D(H_S)$ from macroscopic magnetization reversal type measurements, none of them is completely satisfactory for all levels of intergranular interactions.[1–8]

The recently developed $\Delta H(M,\Delta M)$ method has been used in analyzing and quantifying progress in PRM fabrication.[2,6,8] This method measures the field difference $\Delta H$ at constant magnetization $M$ between the major hysteresis loop and a number of recoil curves, which each start at a certain distance $\Delta M$ away from saturation. Within the mean-field approximation, the functional dependency of $\Delta H$ can be written as $\Delta H(M,\Delta M)=I^{-1}[(1-M)/2]-I^{-1}[(1-M-\Delta M)/2]$ where $I^{-1}$ is the inverse of the integral $I(x)=\int_{-\infty}^{x}D(H_S)dH_S$. Within the framework of this method, $\Delta H$ is independent from the grain interactions, which allows for a direct experimental access to determining $D(H_S)$. For certain parameterized distribution functions, one can derive analytic expressions for $\Delta H$. For example, for a Gaussian distribution of width $\sigma$, one finds $\Delta H_G(M,\Delta M)=\sqrt{2}\sigma[\mathrm{erf}^{-1}(M+\Delta M)-\mathrm{erf}^{-1}(M)]$. Details of this method and the analysis formalism have been previously described.[2,6,8,9]

The $\Delta H(M,\Delta M)$ method was demonstrated to have several advantages over comparable methods. First, it allows the determination of the entire $D(H_S)$ distribution and its functional form and not just a single characteristic parameter.[2,6] Second, it has a well-defined reliability range and it allows for oversampling, which makes self-consistency checks feasible.[9] Third, its failure mode was found to show universal behavior, independent from the detailed lattice structure in numerical simulations.[10]

Despite these advantages, it is still unknown whether the reliability range of the $\Delta H(M,\Delta M)$ method with respect to exchange interactions might be affected by the simultaneous presence of dipolar interactions. Given the fact that this simultaneous presence of both interactions is the realistic case for actual recording media, it is an important issue, which we have studied in this letter.

For our numerical studies, we model each media grain as a symmetric hysteron, which generates a rectangular hysteresis loop in an applied field $H$.[9] The half width of the hysteresis loop is just the intrinsic switching field $H_S$ of this hysteron. We further assume that the magnetization of each hysteron exhibits values of $\pm 1$ only and orients exactly along the applied field $H$ and perpendicular to the recording layer. The PRM is then represented by a square or triangular lattice of symmetric hysterons with periodic boundary conditions. The model Hamiltonian can then be written as $\mathcal{H}=-J_{\mathrm{ex}}\Sigma_{\langle i,j\rangle}S_iS_j+J_{\mathrm{dp}}\Sigma_{i\neq j}S_iS_j/d_{ij}^3-\Sigma_i[H+\mathrm{sgn}(S_i)H_{S_i}]S_i$. Here, the first term represents that hysterons interact ferromagnetically with their nearest neighbors by means of exchange interactions of strength $J_{\mathrm{ex}}$. The second term represents that hysterons exhibit a distance-dependent and antiferromagnetic (AFM) dipolar interaction of strength $J_{\mathrm{dp}}$ with all other hysterons. Here, $d_{ij}$ denotes the distance between hysteron $i$ and $j$ in lattice units.[11] Note that due to our restricted geometry with perfectly aligned perpendicular magnetization (representing high-anisotropy materials), dipolar effects cause an effective interaction that is AFM in its nature. The third term accounts for the effect of the external field and the intrinsic switching field. This model is referred to as the *interacting random hysteron model*. The algorithm for the simulation of the major hysteresis loops and the corresponding recoil curves of this model is described in Ref. 9. For the numerical calculations of long-range dipolar interactions in a system with periodic boundary conditions, we utilized the efficient formalism described by Lekner.[12]

For our numerical study of the $\Delta H(M,\Delta M)$ method's reliability, we assume a Gaussian distribution $D(H_S)$ of width $\sigma$ for a two-dimensional square lattice comprising of total $N$ hysterons. Different system sizes ranging from $50^2$ to $400^2$ have been studied to estimate finite-size inaccuracies. Re-

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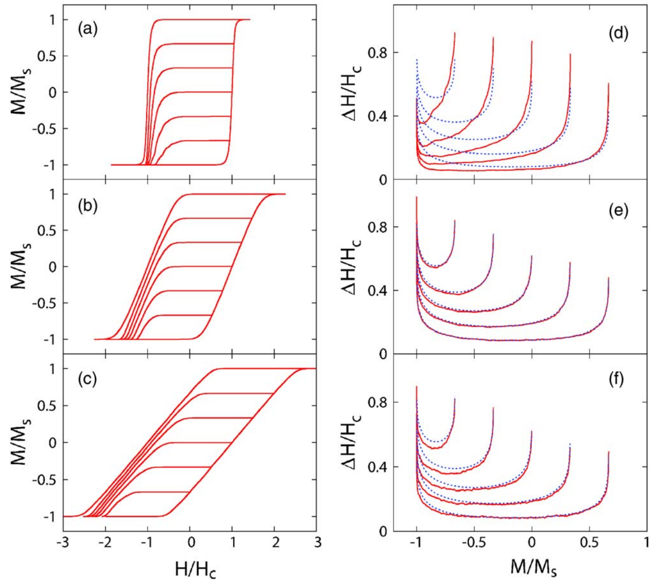

FIG. 1. (Color online) Using Gaussian $D(H_S)$ with width $\sigma=1.0$ to calculate the $M(H)$ curves and $\Delta H(M,\Delta M)$ curves on a two-dimensional square lattice with $N=100^2$ hysterons and $J_{ex}=0.4$. [(a) and (d)] $J_{dp}=0$. [(b) and (e)] $J_{dp}=0.4$. [(c) and (f)] $J_{dp}=0.8$. (Left) $M(H)$ curves: main loop and five recoil curves. (Right) $\Delta H(M,\Delta M)$ curves for the five recoil curves: (solid lines) numerical result; (dotted lines) mean-field approximation. Here $M$ (or $\Delta M$) is normalized to the saturation value $M_S=N$ and $H$ (or $\Delta H$) is normalized to the coercive field $H_C$.

sults presented here were calculated for $100^2$, which we found to be sufficiently precise in all cases. In our model Hamiltonian, $J_{ex}$, $J_{dp}$, $H$, $H_S$, and $\sigma$ all have dimensions of energy. We set $\sigma=1$ to be the unit of energy. We vary both $J_{ex}$ and $J_{dp}$. For each parameter set $(J_{ex},J_{dp})$, we calculate the complete set of $M(H)$ curves (both the saturation hysteresis loop and recoil curves), from which $\Delta H(M,\Delta M)$ data sets are then extracted.

The results displayed in Fig. 1 show several specific examples for parameter sets $(J_{ex},J_{dp})=(0.4,0)$, (0.4,0.4), and (0.4,0.8). The simulated $M(H)$ curves are shown in the left column. It is clearly seen that increasing the strength of dipolar interactions substantially shears the hysteresis loops as expected. The right column displays the corresponding $\Delta H(M,\Delta M)$ curves. The solid lines are the numerically extracted results from the simulated $M(H)$ curves while the dotted lines denote the mean-field behavior according to the expression of $\Delta H_G(M,\Delta M)$. Comparing Figs. 1(d) and 1(e), we find that dipolar interactions of intermediate strength make the system most mean field like.

To study the effect of dipolar interactions on the $\Delta H(M,\Delta M)$ method's reliability in a more systematic and quantitative way, we need to introduce quantitative reliability measures. Obviously, the reliability range of the mean-field approximation, upon which the $\Delta H(M,\Delta M)$ method is based, can be checked by means of a least-squares fit of $\Delta H_G(M,\Delta M)$ to the numerical data and a subsequent analysis of the conventional fit-quality measures, such as (1) the square of the multiple correlation coefficient $R^2$ and (2) the percentage difference $P_d$ between the fitting result and the input parameter $\sigma$. By definition, $R^2=1$ and $P_d=0$ would correspond to perfect data fitting, i.e., the exactness of the mean-field limit. We therefore calculated a least-squares fit to $\Delta H_G(M,\Delta M)$ for the numerically extracted $\Delta H(M,\Delta M)$ data for each parameter set of $(J_{ex},J_{dp})$. From these fits, we then computed both $P_d$ and $R^2$. The results are shown in contour plots (see Fig. 2).

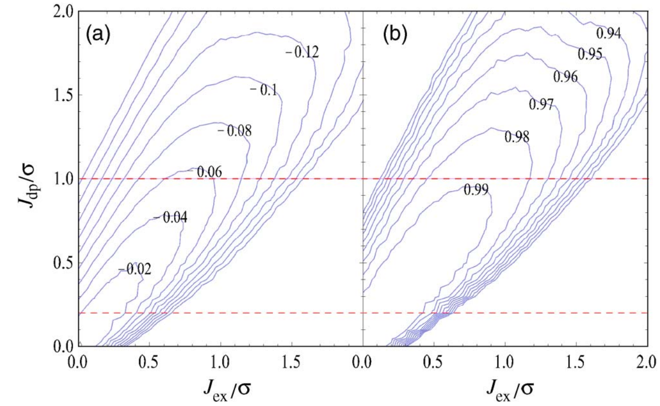

FIG. 2. (Color online) Contour plots of the fit quality measures as functions of $J_{ex}$ and $J_{dp}$, both in units of $\sigma$. (a) $P_d$. (b) $R^2$. Dotted lines indicate the range of $J_{dp}/\sigma$ for realistic materials.

The shape of the contour plots is rather interesting. First, it is nearly symmetric along the diagonal direction, i.e., $J_{dp}/J_{ex}=1$. This clearly demonstrates that the roles of exchange and dipolar interactions in determining $D(H_S)$ are almost equally important. Individually increasing either one will make the $\Delta H(M,\Delta M)$ method less reliable, while increasing both of them with proper strength ratio of order 1 will substantially extend the reliability range. Second, the shape is not really symmetric. It is tilted upwards and smoother on the high $J_{dp}$ side than the high $J_{ex}$ side. This suggests that the $\Delta H(M,\Delta M)$ method can clearly cope with higher dipolar interactions than exchange interactions, in agreement with previous micromagnetic tests.[6]

The overall shape of the contours can be qualitatively explained by the *interaction compensation effect*. From the model Hamiltonian, we know that the intergranular exchange interactions are ferromagnetic (FM) and short range while the dipolar interactions are AFM and long range. The competition between the two "opposite" interaction tendencies will yield a variety of system behaviors. Generally speaking, as we steadily increase $J_{ex}$ from 0 to higher values while keeping $J_{dp}$ constant, we shift the system from the AFM-interaction-dominated regime to the mean-field regime and to the FM-interaction-dominated regime. Only within the interaction compensation region, the FM and AFM interaction tendencies nearly cancel each other. Consequently, the system is most mean field like and the $\Delta H(M,\Delta M)$ method becomes most reliable there. From the model Hamiltonian, we also notice that if $J_{ex}/J_{dp}=1$, the exchange and dipolar interactions will cancel exactly for the nearest-neighboring hysterons which have $d_{ij}=1$ in the lattice units. However, due to the long-range and distance-dependent features of dipolar interactions, this cancellation will not be exact for hysterons with longer distances. This explains why the interaction compensation region is only roughly symmetric along the diagonal direction.

To quantify the interaction compensation region or equivalently the reliability range of the $\Delta H(M,\Delta M)$ method, one can define a critical value for each reliability measure, above which this method is sufficiently accurate. For example, we might define $R_c^2=0.98$ to be the critical value for $R^2$ in accordance with the best available experimental data.[6]

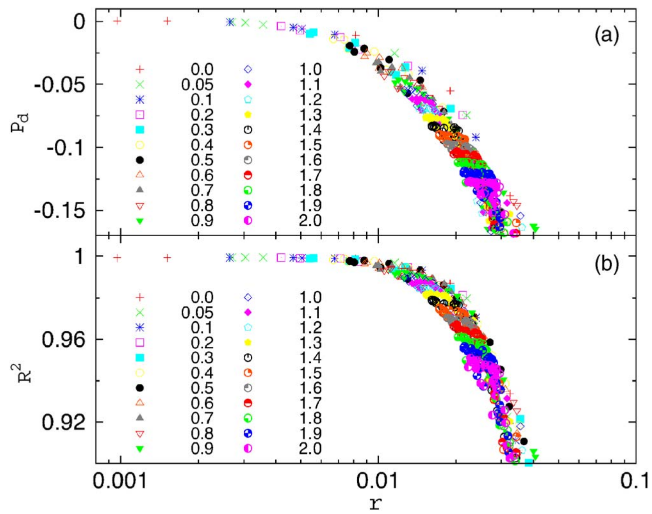


FIG. 3. (Color online) Correlations between the reliability measures: (a) $P_d$ and $r$; (b) $R^2$ and $r$. Each data point represents a different parameter set $(J_{ex}, J_{dp})$. Data points with the same $J_{dp}$ values are grouped and shown with the same point symbol, as indicated by the legend.

Then the reliability range for the parameter set $(J_{ex}, J_{dp})$ can be clearly seen from the region enclosed by the second highest contour in the $R^2$ plot. This particular contour will be referred to as the critical contour, inside which the system is virtually mean field like and the $\Delta H(M,\Delta M)$ method is reliable. Note that for practical recording media, the ratio of $J_{dp}/\sigma$ will probably be limited within the range of 0.2–1, which happens to have an overlap with the critical contour (see Fig. 2). One can then easily see that within the realistic $J_{dp}/\sigma$ range, the dipolar interactions improve the reliability range of the $\Delta H(M,\Delta M)$ method up to higher $J_{ex}$ values. We also notice that for $J_{ex}/\sigma \leq 0.2$ (read off from the intercept of the $R^2$ critical contour on the $J_{ex}$ axis), higher dipolar interaction will only make the $\Delta H(M,\Delta M)$ method worse. However, this part is very small compared to the part where dipolar interactions make the $\Delta H(M,\Delta M)$ method robust.

Besides the fit-quality measures $R^2$ and $P_d$, there is a self-consistency-check measure, which is based upon data redundancy in between multiple recoil curves.[9] One can test data for deviations from this redundancy by means of a quantity $r=1/n\Sigma_{i,j}\langle r_{ij}^2(M)\rangle^{1/2}$ where $r_{ij}(M)$ is by definition identical to zero within the mean-field approximation, so is $r$.[13] The specific advantage of this quantity $r$ is that it can be directly calculated from data sets alone without the need for any data fitting. Therefore, it is important to analyze the possible correlation between the fit-quality measure (either $R^2$ or $P_d$) and the deviation-from-redundancy measure $r$. Knowledge of this correlation will enable us to estimate the suitability of the $\Delta H(M,\Delta M)$ method without any data fitting. Considering this, we calculated $r$ from the numerical $\Delta H(M,\Delta M)$ data for each parameter set of $(J_{ex}, J_{dp})$. Overall, the contour plot of $r$ shows very similar features as of the ones being displayed in Fig. 2 for $R^2$ and $P_d$. To visualize and quantify the correlation between $R^2$ ($P_d$) and $r$, we plot $R^2$ ($P_d$) versus $r$ for the complete set of different $(J_{ex}, J_{dp})$ parameter (see Fig. 3). We find that the data collapse fairly well onto a single line in the high $R^2$ or low $|P_d|$ range, in which the utilization of the $\Delta H(M,\Delta M)$ method is sensible and accurate. This indicates that $R^2$ and $P_d$ are highly correlated with $r$ in the regime where these quantities matter. Due to the knowledge of these correlations, one now has a criterion that enables a judgment on the usefulness and reliability of any $\Delta H(M,\Delta M)$-data set evaluation. For that, one simply determines the $r$ value from experimental or modeling data sets, looks up the expected precision with the help of Fig. 3, and then decides if a further data analysis is warranted or not.

In summary, we find that the presence of dipolar interactions similar in size to those of real PRM makes the $\Delta H(M,\Delta M)$ method substantially more precise and robust. The deviation-from-redundancy measure $r$, which is a self-consistency check, is found to be a good predictor of the $\Delta H(M,\Delta M)$ method's reliability and can be utilized as a criterion to decide if a full scale data analysis is warranted.

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[13] For recoil curve pair $(i,j)$, the deviation from redundancy is defined as $r_{ij}(M)=[\Delta H_i(M)+\Delta H_j(M-\Delta M_j)-\Delta H_i(M-\Delta M_j)-\Delta H_j(M-\Delta M_j+\Delta M_i)]/[\Delta H_i(M)+\Delta H_j(M-\Delta M_j)+\Delta H_i(M-\Delta M_j)+\Delta H_j(M-\Delta M_j+\Delta M_i)]$. In mean field theory, it can be exactly proven that $r_{ij}(M)=0$. For details, see Ref. 9.